

Hazelnut: A Bidirectionally Typed Structure Editor Calculus



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Q: What is it that program editors reason about?

Q: What is it that textual program editors reason about?

```
fun summary_stats(m : matrix) =  
  { mean    = stats.mean(m, ColumnWise),  
    std     = stats.std(m,  
    median =
```

syntactically malformed program text

Syntactic error recovery heuristics

```
fun summary_stats(m : matrix) =  
  { mean    = stats.mean(m, ColumnWise),  
    std     = stats.std(m, □),  
    median = □ }
```

syntactically malformed program text → term with holes

Syntactic structure editors

```
fun summary_stats(m : matrix) =  
  { mean    = stats.mean(m, ColumnWise),  
    std     = stats.std(m, 0),  
    median = 0 }
```

~~syntactically malformed program text~~ → term with holes

[Teitelbaum and Reps, Comm. ACM 1981; many others since]

Q: How to reason statically about terms with holes?

```
fun summary_stats(m : matrix) =  
  { mean    = stats.mean(m, ColumnWise),  
    std     = stats.std(m, 0),  
    median  = 0 }
```

Q: How to reason statically about terms with holes?

What **type** is synthesized for the function as a whole?



```
fun summary_stats(m : matrix) =  
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Q: How to reason statically about terms with holes?

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```
fun summary_stats(m : matrix) =  
  { mean    = stats.mean(m, ColumnWise),  
    std     = stats.std(m, 0),  
    median  = 0 }
```

```
matrix →  
{ mean    : vec,  
  std     : vec,  
  median  : 0 }
```

Q: How to reason statically about terms with type errors?

What **type** is synthesized for the function as a whole?



```
fun summary_stats(m : matrix) =  
  { mean    = stats.mean(m, ColumnWise),  
    std     = stats.std(m, "oops"),  
    median = □ }
```

Q: How to reason statically about terms with type errors?

What **type** is synthesized for the function as a whole?

↓

```
fun summary_stats(m : matrix) =  
  { mean    = stats.mean(m, ColumnWise),  
    std     = stats.std(m, "oops"),  
    median = □ }
```

↑

Reify type inconsistencies as non-empty holes!

Q: How to reason statically about terms with type errors?

What **type** is synthesized for the function as a whole?



```
fun summary_stats(m : matrix) =  
  { mean    = stats.mean(m, ColumnWise),  
    std     = stats.std(m, "oops"),  
    median  = [] }
```



Reify type inconsistencies as non-empty holes!

```
matrix →  
{ mean    : vec,  
  std     : vec,  
  median  : [] }
```

Contribution 1: A static semantics for lambda terms with holes

HTyp $\dot{\tau} ::= (\dot{\tau} \rightarrow \dot{\tau}) \mid \mathbf{num} \mid \langle \rangle$
HExp $\dot{e} ::= x \mid (\lambda x. \dot{e}) \mid \dot{e}(\dot{e}) \mid \underline{n} \mid (\dot{e} + \dot{e}) \mid \dot{e} : \dot{\tau} \mid \langle \rangle \mid \langle \dot{e} \rangle$

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$\boxed{\dot{\Gamma} \vdash \dot{e} \Rightarrow \dot{\tau}}$ \dot{e} synthesizes $\dot{\tau}$

$\boxed{\dot{\Gamma} \vdash \dot{e} \Leftarrow \dot{\tau}}$ \dot{e} analyzes against $\dot{\tau}$

Contribution 1: A static semantics for lambda terms with holes

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$\boxed{\dot{\Gamma} \vdash \dot{e} \Rightarrow \dot{\tau}}$ \dot{e} synthesizes $\dot{\tau}$

$\boxed{\dot{\Gamma} \vdash \dot{e} \Leftarrow \dot{\tau}}$ \dot{e} analyzes against $\dot{\tau}$

...

$\overline{\dot{\Gamma} \vdash \langle \rangle \Rightarrow \langle \rangle}$

$\frac{\dot{\Gamma} \vdash \dot{e} \Rightarrow \dot{\tau}}{\dot{\Gamma} \vdash \langle \dot{e} \rangle \Rightarrow \langle \rangle}$

Contribution 1: A static semantics for lambda terms with holes

HTyp $\dot{\tau} ::= (\dot{\tau} \rightarrow \dot{\tau}) \mid \mathbf{num} \mid \mathbb{O}$

HExp $\dot{e} ::= x \mid (\lambda x. \dot{e}) \mid \dot{e}(\dot{e}) \mid \underline{n} \mid (\dot{e} + \dot{e}) \mid \dot{e} : \dot{\tau} \mid \mathbb{O} \mid \mathbb{O}(\dot{e})$

$\boxed{\dot{\Gamma} \vdash \dot{e} \Rightarrow \dot{\tau}}$ \dot{e} synthesizes $\dot{\tau}$

$\boxed{\dot{\Gamma} \vdash \dot{e} \Leftarrow \dot{\tau}}$ \dot{e} analyzes against $\dot{\tau}$

...

$\overline{\dot{\Gamma} \vdash \mathbb{O} \Rightarrow \mathbb{O}}$

$\frac{\dot{\Gamma} \vdash \dot{e} \Rightarrow \dot{\tau}}{\dot{\Gamma} \vdash \mathbb{O}(\dot{e}) \Rightarrow \mathbb{O}}$

...

$\frac{\dot{\Gamma} \vdash \dot{e} \Rightarrow \dot{\tau}' \quad \dot{\tau} \sim \dot{\tau}'}{\dot{\Gamma} \vdash \dot{e} \Leftarrow \dot{\tau}}$

Contribution 1: A static semantics for lambda terms with holes

HTyp $\dot{\tau} ::= (\dot{\tau} \rightarrow \dot{\tau}) \mid \mathbf{num} \mid \mathbb{O}$

HExp $\dot{e} ::= x \mid (\lambda x. \dot{e}) \mid \dot{e}(\dot{e}) \mid \underline{n} \mid (\dot{e} + \dot{e}) \mid \dot{e} : \dot{\tau} \mid \mathbb{O} \mid \mathbb{O}(\dot{e})$

$\boxed{\dot{\Gamma} \vdash \dot{e} \Rightarrow \dot{\tau}}$ \dot{e} synthesizes $\dot{\tau}$

$\boxed{\dot{\Gamma} \vdash \dot{e} \Leftarrow \dot{\tau}}$ \dot{e} analyzes against $\dot{\tau}$

...

...

$\overline{\dot{\Gamma} \vdash \mathbb{O} \Rightarrow \mathbb{O}}$

$\frac{\dot{\Gamma} \vdash \dot{e} \Rightarrow \dot{\tau}' \quad \dot{\tau} \sim \dot{\tau}'}{\dot{\Gamma} \vdash \dot{e} \Leftarrow \dot{\tau}}$

$\frac{\dot{\Gamma} \vdash \dot{e} \Rightarrow \dot{\tau}}{\dot{\Gamma} \vdash \mathbb{O}(\dot{e}) \Rightarrow \mathbb{O}}$

$\boxed{\dot{\tau} \sim \dot{\tau}'}$ $\dot{\tau}$ and $\dot{\tau}'$ are consistent

$\overline{\mathbb{O} \sim \dot{\tau}} \quad \overline{\dot{\tau} \sim \mathbb{O}} \quad \overline{\dot{\tau} \sim \dot{\tau}} \quad \frac{\dot{\tau}_1 \sim \dot{\tau}'_1 \quad \dot{\tau}_2 \sim \dot{\tau}'_2}{(\dot{\tau}_1 \rightarrow \dot{\tau}_2) \sim (\dot{\tau}'_1 \rightarrow \dot{\tau}'_2)}$

Contribution 1: A static semantics for lambda terms with holes

HTyp $\dot{\tau} ::= (\dot{\tau} \rightarrow \dot{\tau}) \mid \text{num} \mid \text{()}$

HExp $\dot{e} ::= x \mid (\lambda x. \dot{e}) \mid \dot{e}(\dot{e}) \mid \underline{n} \mid (\dot{e} + \dot{e}) \mid \dot{e} : \dot{\tau} \mid \text{()}$ $\mid \text{()}$

$\boxed{\dot{\Gamma} \vdash \dot{e} \Rightarrow \dot{\tau}}$ \dot{e} synthesizes $\dot{\tau}$

$\boxed{\dot{\Gamma} \vdash \dot{e} \Leftarrow \dot{\tau}}$ \dot{e} analyzes against $\dot{\tau}$

...

$\overline{\dot{\Gamma} \vdash \text{()}} \Rightarrow \text{()}$

$\overline{\dot{\Gamma} \vdash \dot{e} \Rightarrow \dot{\tau}} \Rightarrow \text{()}$

...

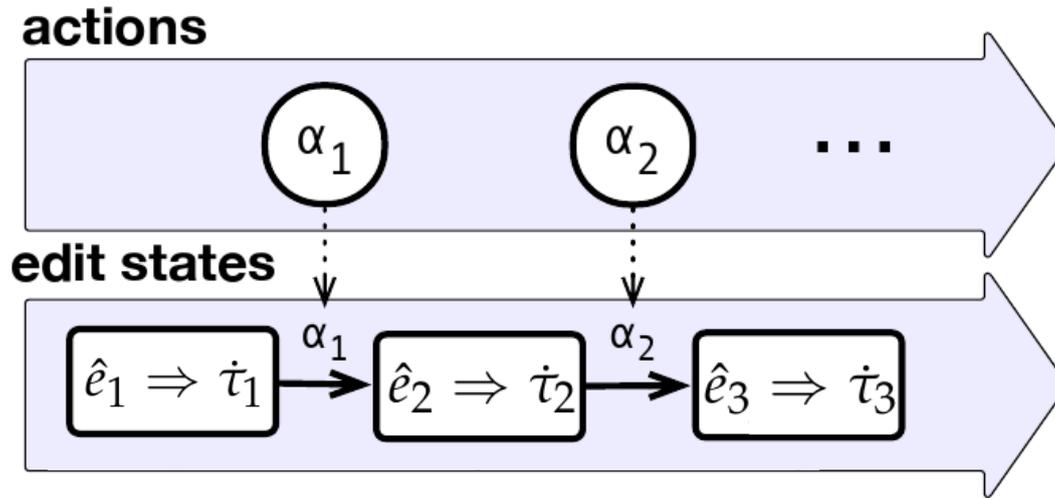
$\frac{\dot{\Gamma} \vdash \dot{e} \Rightarrow \dot{\tau}' \quad \dot{\tau} \sim \dot{\tau}'}{\dot{\Gamma} \vdash \dot{e} \Leftarrow \dot{\tau}}$

$\boxed{\dot{\tau} \sim \dot{\tau}'}$ $\dot{\tau}$ and $\dot{\tau}'$ are consistent

$\overline{\text{()}} \sim \dot{\tau} \quad \overline{\dot{\tau}} \sim \text{()}$ $\overline{\dot{\tau}} \sim \dot{\tau}$ $\frac{\dot{\tau}_1 \sim \dot{\tau}'_1 \quad \dot{\tau}_2 \sim \dot{\tau}'_2}{(\dot{\tau}_1 \rightarrow \dot{\tau}_2) \sim (\dot{\tau}'_1 \rightarrow \dot{\tau}'_2)}$

coincides with **gradual typing**
[Siek and Taha, 2006]

Contribution 2: A typed edit action semantics



See <http://hazeltrove.org/>

Contribution 2: A typed edit action semantics

$$\begin{aligned} \text{ZTyp } \hat{\tau} &::= \triangleright \hat{\tau} \triangleleft \mid (\hat{\tau} \rightarrow \hat{\tau}) \mid (\hat{\tau} \rightarrow \hat{\tau}) \\ \text{ZExp } \hat{e} &::= \triangleright \hat{e} \triangleleft \mid (\lambda x. \hat{e}) \mid \hat{e}(\hat{e}) \mid \hat{e}(\hat{e}) \mid (\hat{e} + \hat{e}) \mid (\hat{e} + \hat{e}) \\ &\mid \hat{e} : \hat{\tau} \mid \hat{e} : \hat{\tau} \mid (\hat{e}) \end{aligned}$$

Contribution 2: A typed edit action semantics

ZTyp $\hat{\tau} ::= \triangleright \hat{\tau} \triangleleft \mid (\hat{\tau} \rightarrow \hat{\tau}) \mid (\hat{\tau} \rightarrow \hat{\tau})$

ZExp $\hat{e} ::= \triangleright \hat{e} \triangleleft \mid (\lambda x. \hat{e}) \mid \hat{e}(\hat{e}) \mid \hat{e}(\hat{e}) \mid (\hat{e} + \hat{e}) \mid (\hat{e} + \hat{e})$
 $\mid \hat{e} : \hat{\tau} \mid \hat{e} : \hat{\tau} \mid (\hat{e})$

Action $\alpha ::= \text{move } \delta \mid \text{construct } \psi \mid \text{del} \mid \text{finish}$

Dir $\delta ::= \text{child } n \mid \text{parent}$

Shape $\psi ::= \text{arrow} \mid \text{num}$
 $\mid \text{asc} \mid \text{var } x \mid \text{lam } x \mid \text{ap} \mid \text{lit } n \mid \text{plus}$

Contribution 2: A typed edit action semantics

ZTyp $\hat{\tau} ::= \triangleright \hat{\tau} \triangleleft \mid (\hat{\tau} \rightarrow \hat{\tau}) \mid (\hat{\tau} \rightarrow \hat{\tau})$
ZExp $\hat{e} ::= \triangleright \hat{e} \triangleleft \mid (\lambda x. \hat{e}) \mid \hat{e}(\hat{e}) \mid \hat{e}(\hat{e}) \mid (\hat{e} + \hat{e}) \mid (\hat{e} + \hat{e})$
| $\hat{e} : \hat{\tau} \mid \hat{e} : \hat{\tau} \mid (\hat{e})$

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| $\text{asc} \mid \text{var } x \mid \text{lam } x \mid \text{ap} \mid \text{lit } n \mid \text{plus}$

$$\dot{\Gamma} \vdash \hat{e} \Rightarrow \hat{\tau} \xrightarrow{\alpha} \hat{e}' \Rightarrow \hat{\tau}'$$

$$\dot{\Gamma} \vdash \hat{e} \xrightarrow{\alpha} \hat{e}' \Leftarrow \hat{\tau}$$

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Action $\alpha ::= \text{move } \delta \mid \text{construct } \psi \mid \text{del} \mid \text{finish}$
Dir $\delta ::= \text{child } n \mid \text{parent}$
Shape $\psi ::= \text{arrow} \mid \text{num}$
| $\text{asc} \mid \text{var } x \mid \text{lam } x \mid \text{ap} \mid \text{lit } n \mid \text{plus}$

$$\boxed{\dot{\Gamma} \vdash \hat{e} \Rightarrow \hat{\tau} \xrightarrow{\alpha} \hat{e}' \Rightarrow \hat{\tau}'}$$

$$\boxed{\dot{\Gamma} \vdash \hat{e} \xrightarrow{\alpha} \hat{e}' \Leftarrow \hat{\tau}}$$

$$\frac{\dot{\Gamma} \vdash \triangleright (\emptyset) \triangleleft \Rightarrow (\emptyset)}{\dot{\Gamma} \vdash \triangleright (\emptyset) \triangleleft \Rightarrow (\emptyset) \xrightarrow{\text{construct lit } n} \triangleright \underline{n} \triangleleft \Rightarrow \text{num}}$$

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Action $\alpha ::= \text{move } \delta \mid \text{construct } \psi \mid \text{del} \mid \text{finish}$
Dir $\delta ::= \text{child } n \mid \text{parent}$
Shape $\psi ::= \text{arrow} \mid \text{num}$
| $\text{asc} \mid \text{var } x \mid \text{lam } x \mid \text{ap} \mid \text{lit } n \mid \text{plus}$

$$\boxed{\dot{\Gamma} \vdash \hat{e} \Rightarrow \hat{\tau} \xrightarrow{\alpha} \hat{e}' \Rightarrow \hat{\tau}'}$$

$$\boxed{\dot{\Gamma} \vdash \hat{e} \xrightarrow{\alpha} \hat{e}' \Leftarrow \hat{\tau}}$$

$$\frac{\dot{\Gamma} \vdash \triangleright (\emptyset) \triangleleft \Rightarrow (\emptyset)}{\dot{\Gamma} \vdash \triangleright (\emptyset) \triangleleft \xrightarrow{\text{construct lit } n} \triangleright \underline{n} \triangleleft \Rightarrow \text{num}}$$

$$\frac{\dot{\Gamma} \vdash \hat{e} \xrightarrow{\alpha} \hat{e}' \Leftarrow \text{num}}{\dot{\Gamma} \vdash (\hat{e} + \hat{e}) \Rightarrow \text{num} \xrightarrow{\alpha} (\hat{e}' + \hat{e}) \Rightarrow \text{num}}$$

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 ZExp $\hat{e} ::= \triangleright \hat{e} \triangleleft \mid (\lambda x. \hat{e}) \mid \hat{e}(\hat{e}) \mid \hat{e}(\hat{e}) \mid (\hat{e} + \hat{e}) \mid (\hat{e} + \hat{e})$
 $\mid \hat{e} : \hat{\tau} \mid \hat{e} : \hat{\tau} \mid (\hat{e})$

Action $\alpha ::= \text{move } \delta \mid \text{construct } \psi \mid \text{del} \mid \text{finish}$
 Dir $\delta ::= \text{child } n \mid \text{parent}$
 Shape $\psi ::= \text{arrow} \mid \text{num}$
 $\mid \text{asc} \mid \text{var } x \mid \text{lam } x \mid \text{ap} \mid \text{lit } n \mid \text{plus}$

$$\boxed{\dot{\Gamma} \vdash \hat{e} \Rightarrow \hat{\tau} \xrightarrow{\alpha} \hat{e}' \Rightarrow \hat{\tau}'}$$

$$\boxed{\dot{\Gamma} \vdash \hat{e} \xrightarrow{\alpha} \hat{e}' \Leftarrow \hat{\tau}}$$

$$\frac{\dot{\Gamma} \vdash \triangleright (\emptyset) \triangleleft \Rightarrow (\emptyset) \quad \text{construct lit } n}{\dot{\Gamma} \vdash \triangleright \underline{n} \triangleleft \Rightarrow \text{num}}$$

$$\frac{\dot{\Gamma} \vdash \hat{e}^\diamond \Rightarrow \hat{\tau}' \quad \dot{\Gamma} \vdash \hat{e} \Rightarrow \hat{\tau}' \xrightarrow{\alpha} \hat{e}' \Rightarrow \hat{\tau}'' \quad \hat{\tau} \sim \hat{\tau}''}{\dot{\Gamma} \vdash \hat{e} \xrightarrow{\alpha} \hat{e}' \Leftarrow \hat{\tau}}$$

$$\dot{\Gamma} \vdash \hat{e} \xrightarrow{\alpha} \hat{e}' \Leftarrow \text{num}$$

$$\frac{}{\dot{\Gamma} \vdash (\hat{e} + \hat{e}) \Rightarrow \text{num} \xrightarrow{\alpha} (\hat{e}' + \hat{e}) \Rightarrow \text{num}}$$

Contribution 2: A typed edit action semantics

ZTyp $\hat{\tau} ::= \triangleright \hat{\tau} \triangleleft \mid (\hat{\tau} \rightarrow \hat{\tau}) \mid (\hat{\tau} \rightarrow \hat{\tau})$
 ZExp $\hat{e} ::= \triangleright \hat{e} \triangleleft \mid (\lambda x. \hat{e}) \mid \hat{e}(\hat{e}) \mid \hat{e}(\hat{e}) \mid (\hat{e} + \hat{e}) \mid (\hat{e} + \hat{e})$
 $\mid \hat{e} : \hat{\tau} \mid \hat{e} : \hat{\tau} \mid (\hat{e})$

Action $\alpha ::= \text{move } \delta \mid \text{construct } \psi \mid \text{del} \mid \text{finish}$
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 $\mid \text{asc} \mid \text{var } x \mid \text{lam } x \mid \text{ap} \mid \text{lit } n \mid \text{plus}$

$$\boxed{\dot{\Gamma} \vdash \hat{e} \Rightarrow \hat{\tau} \xrightarrow{\alpha} \hat{e}' \Rightarrow \hat{\tau}'}$$

$$\boxed{\dot{\Gamma} \vdash \hat{e} \xrightarrow{\alpha} \hat{e}' \Leftarrow \hat{\tau}}$$

$$\frac{\dot{\Gamma} \vdash \triangleright (\emptyset) \triangleleft \Rightarrow (\emptyset)}{\dot{\Gamma} \vdash \triangleright (\emptyset) \triangleleft \xrightarrow{\text{construct lit } n} \triangleright \underline{n} \triangleleft \Rightarrow \text{num}}$$

$$\frac{\dot{\Gamma} \vdash \hat{e} \xrightarrow{\alpha} \hat{e}' \Leftarrow \text{num}}{\dot{\Gamma} \vdash (\hat{e} + \hat{e}) \Rightarrow \text{num} \xrightarrow{\alpha} (\hat{e}' + \hat{e}) \Rightarrow \text{num}}$$

$$\frac{\dot{\Gamma} \vdash \hat{e}^\diamond \Rightarrow \hat{\tau}' \quad \dot{\Gamma} \vdash \hat{e} \Rightarrow \hat{\tau}' \xrightarrow{\alpha} \hat{e}' \Rightarrow \hat{\tau}'' \quad \hat{\tau} \sim \hat{\tau}''}{\dot{\Gamma} \vdash \hat{e} \xrightarrow{\alpha} \hat{e}' \Leftarrow \hat{\tau}}$$

$$\frac{\hat{\tau} \approx \text{num}}{\dot{\Gamma} \vdash \triangleright (\emptyset) \triangleleft \xrightarrow{\text{construct lit } n} (\triangleright \underline{n} \triangleleft) \Leftarrow \hat{\tau}}$$

Metatheorem: Sensibility

Every edit action leaves the edit state well-typed.

Theorem 1 (Action Sensibility).

1. If $\dot{\Gamma} \vdash \hat{e}^\diamond \Rightarrow \dot{\tau}$ and $\dot{\Gamma} \vdash \hat{e} \Rightarrow \dot{\tau} \xrightarrow{\alpha} \hat{e}' \Rightarrow \dot{\tau}'$ then $\dot{\Gamma} \vdash \hat{e}'^\diamond \Rightarrow \dot{\tau}'$.
2. If $\dot{\Gamma} \vdash \hat{e}^\diamond \Leftarrow \dot{\tau}$ and $\dot{\Gamma} \vdash \hat{e} \xrightarrow{\alpha} \hat{e}' \Leftarrow \dot{\tau}$ then $\dot{\Gamma} \vdash \hat{e}'^\diamond \Leftarrow \dot{\tau}$.

Metatheorem: Reachability

The cursor can reach any position in the program.

Theorem 3 (Reachability).

1. If $\hat{\tau}^\diamond = \hat{\tau}'^\diamond$ then there exists some $\bar{\alpha}$ such that $\bar{\alpha}$ movements and $\hat{\tau} \xrightarrow{\bar{\alpha}}^* \hat{\tau}'$.
2. If $\dot{\Gamma} \vdash \hat{e}^\diamond \Rightarrow \dot{\tau}$ and $\hat{e}^\diamond = \hat{e}'^\diamond$ then there exists some $\bar{\alpha}$ such that $\bar{\alpha}$ movements and $\dot{\Gamma} \vdash \hat{e} \Rightarrow \dot{\tau} \xrightarrow{\bar{\alpha}}^* \hat{e}' \Rightarrow \dot{\tau}$.
3. If $\dot{\Gamma} \vdash \hat{e}^\diamond \Leftarrow \dot{\tau}$ and $\hat{e}^\diamond = \hat{e}'^\diamond$ then there exists some $\bar{\alpha}$ such that $\bar{\alpha}$ movements and $\dot{\Gamma} \vdash \hat{e} \xrightarrow{\bar{\alpha}}^* \hat{e}' \Leftarrow \dot{\tau}$.

Metatheorem: Constructability

Any well-typed expression can be constructed using edit actions.

Theorem 6 (Constructability).

1. For every $\dot{\tau}$ there exists $\bar{\alpha}$ such that $\triangleright(\emptyset)\triangleleft \xrightarrow{\bar{\alpha}}^* \triangleright\dot{\tau}\triangleleft$.
2. If $\dot{\Gamma} \vdash \dot{e} \Rightarrow \dot{\tau}$ then there exists $\bar{\alpha}$ such that:

$$\dot{\Gamma} \vdash \triangleright(\emptyset)\triangleleft \Rightarrow (\emptyset) \xrightarrow{\bar{\alpha}}^* \triangleright\dot{e}\triangleleft \Rightarrow \dot{\tau}$$

3. If $\dot{\Gamma} \vdash \dot{e} \Leftarrow \dot{\tau}$ then there exists $\bar{\alpha}$ such that:

$$\dot{\Gamma} \vdash \triangleright(\emptyset)\triangleleft \xrightarrow{\bar{\alpha}}^* \triangleright\dot{e}\triangleleft \Leftarrow \dot{\tau}$$

Summary: Hazelnut

- A **static semantics** for terms with holes and type inconsistencies.
- An **typed action semantics** that maintains sensibility invariant.
 - **HZ**: A reference implementation written in OCaml React + js_of_ocaml.
- A **rich metatheory** that establishes the correctness of Hazelnut.
 - Mechanized using the **Agda** proof assistant.
 - Guides the definition of an extension (sum types – see paper!)

From Hazelnut to Hazel



Hazel

Navigation menu with buttons: Numerics, Plotting, Statistics, and a plus sign (+).

```
fun summary_stats(m : matrix<float>)  
  { mean = mean(m, ColumnWise)  
    std  = std(m, □)  
    median = □ } (a)
```

```
let my_data : matrix<float> =  
  [ 1.1 | 2.3 | 3.0 | 4.1 | 5.2 ] (b)  
  [ 1.2 | 1.8 | 3.1 | 4.1 | 5.2 ]  
  [ 0.9 | 2.2 | 2.7 | 3.5 | 4.9 ]  
  [ 0.8 | 1.5 | 3.3 | 4.3 | 4.7 ]
```

```
summary_stats(my_data)  
  
{ mean = [1.0 | 2.0 | 3.0 | 4.0 | 5.0]  
  std  = std(my_data, □)  
  median = □ } (c)
```

Type at cursor: dimension

Action search... (d)

- ColumnWise (most probable)
- RowWise
- Factor to variable...
- (□)
- Full action palette...

From Hazelnut to Hazel



Hazel

Numerics ▾ Plotting ▾ Statistics ▾ +

```
fun summary_stats(m : matrix<float>)  
  { mean = mean(m, ColumnWise)  
    std  = std(m, □)  
    median = □ }
```

(a)

TODO: scale up POPL17

```
let my_data : matrix<float> =  
  [ [ 1.1 | 2.3 | 3.0 | 4.1 | 5.2 ]  
    [ 1.2 | 1.8 | 3.1 | 4.1 | 5.2 ]  
    [ 0.9 | 2.2 | 2.7 | 3.5 | 4.9 ]  
    [ 0.8 | 1.5 | 3.3 | 4.3 | 4.7 ] ]
```

(b)

```
summary_stats(my_data)  
  
{ mean = [ 1.0 | 2.0 | 3.0 | 4.0 | 5.0 ]  
  std  = std(my_data, □)  
  median = □ }
```

(c)

Type at cursor: dimension

Action search... (d)

- ColumnWise (most probable)
- RowWise
- Factor to variable...
- (□)
- Full action palette...

From Hazelnut to Hazel



Hazel

Numerics ▾

Plotting ▾

Statistics ▾

+

```
fun summary_stats(m : matrix<float>)  
  { mean = mean(m, ColumnWise)  
    std  = std(m, □)  
    median = □ } (a)
```

```
let my_data : matrix<float> =  
  [ [ 1.1 | 2.3 | 3.0 | 4.1 | 5.2 ]  
    [ 1.2 | 1.8 | 3.1 | 4.1 | 5.2 ]  
    [ 0.9 | 2.2 | 2.7 | 3.5 | 4.9 ]  
    [ 0.8 | 1.5 | 3.3 | 4.3 | 4.7 ] ] (b)
```

```
summary_stats(my_data)  
  
{ mean = [ 1.0 | 2.0 | 3.0 | 4.0 | 5.0 ]  
  std  = std(my_data, □)  
  median = □ } (c)
```

TODO: type-specific projections
(based on my work at ICSE 2012, ECOOP 2014)

Type at cursor: dimension

Action search... (d)

ColumnWise (most probable)

□(□)

Full action palette...

From Hazelnut to Hazel



Hazel

Numerics Plotting Statistics +

```
fun summary_stats(m : matrix<float>)  
  { mean = mean(m, ColumnWise)  
    std  = std(m, □)  
    median = □ } (a)
```

```
let my_data : matrix<float> =  
  [ [ 1.1 | 2.3 | 3.0 | 4.1 | 5.2 ]  
    [ 1.2 | 1.8 | 3.1 | 4.1 | 5.2 ]  
    [ 0.9 | 2.2 | 2.7 | 3.5 | 4.9 ]  
    [ 0.8 | 1.5 | 3.3 | 4.3 | 4.7 ] ] (b)
```

```
summary_stats(my_data)  
  
{ mean = [1.0 | 2.0 | 3.0 | 4.0 | 5.0]  
  std  = std(my_data, □)  
  median = □ } (c)
```

TODO: a dynamic semantics for incomplete programs (very live programming)

Type at cursor: dimension

Action search... (d)

- ColumnWise (most probable)
- RowWise
- Factor to variable...
- (□)
- Full action palette...

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- Plotting ▾
- Statistics ▾
- + ▾

TODO: an action suggestion semantics

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fun summary_stats(m : matrix<float>)  
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let my_data : matrix<float> = 

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|-----|-----|-----|-----|-----|
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| 0.9 | 2.2 | 2.7 | 3.5 | 4.9 |
| 0.8 | 1.5 | 3.3 | 4.3 | 4.7 |

 (b)
```

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summary_stats(my_data)  
  
{ mean = [1.0 | 2.0 | 3.0 | 4.0 | 5.0]  
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Action search... (d)

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```

```
summary_stats(my_data)  
  
{ mean = [1.0 | 2.0 | 3.0 | 4.0 | 5.0] } (c)  
{ std = std(my_data, □) }  
{ median = □ }
```

Type at cursor: dimension

Action search... (d)

ColumnWise (most probable)

□(□)

Full action palette...

TODO: a statistical model of edit actions

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Hazel

Numerics ▾ Plotting ▾ Statistics ▾ +

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(a)

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    [ 0.8 | 1.5 | 3.3 | 4.3 | 4.7 ] ]
```

(b)

```
summary_stats(my_data)
```

```
{ mean = [ 1.0 | 2.0 | 3.0 | 4.0 | 5.0 ] }  
{ std = std(my_data, □) }  
{ median = □ }
```

(c)

Type at cursor: dimension

Action search... (d)

- ColumnWise (most probable)
- RowWise
- Factor to variable...

TODO: library-defined derived actions

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Numerics ▾

Plotting ▾

Statistics ▾

+

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 (b)
```

```
summary_stats(my_data)
```

```
{ mean = [1.0 | 2.0 | 3.0 | 4.0 | 5.0]  
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Type at cursor: dimension

Action search... (d)

ColumnWise (most probable)

RowWise

Factor to variable...

□(□)

Full action palette...

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 (b)
```

```
summary_stats(my_data)  
  
{ mean = [1.0 | 2.0 | 3.0 | 4.0 | 5.0] } (c)  
{ std = std(my_data, □) }  
{ median = □ }
```

Type at cursor: dimension

Action search... (d)

- ColumnWise (most probable)
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See <http://www.hazelgrove.org/>.

Thanks!