

Hazelnut: A Bidirectionally Typed Structure Editor Calculus



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Q: What is it that program editors reason about?

Q: What is it that textual program editors reason about?

```
fun summary_stats(m : matrix) =  
  { mean    = stats.mean(m, ColumnWise),  
    std     = stats.std(m,  
    median =
```

syntactically malformed program text

Syntactic error recovery heuristics

```
fun summary_stats(m : matrix) =  
  { mean    = stats.mean(m, ColumnWise),  
    std     = stats.std(m, □),  
    median = □ }
```

syntactically malformed program text → term with holes

Syntactic structure editors

```
fun summary_stats(m : matrix) =  
  { mean    = stats.mean(m, ColumnWise),  
    std     = stats.std(m, □),  
    median = □ }
```

~~syntactically malformed program text~~ → term with holes

[Teitelbaum and Reps, Comm. ACM 1981; many others since]

Q: How to reason statically about terms with holes?

```
fun summary_stats(m : matrix) =  
  { mean    = stats.mean(m, ColumnWise),  
    std     = stats.std(m, □),  
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```

Q: How to reason statically about terms with holes?

What **type** is synthesized for the function as a whole?



```
fun summary_stats(m : matrix) =  
  { mean    = stats.mean(m, ColumnWise),  
    std     = stats.std(m, 0),  
    median  = 0 }
```

Q: How to reason statically about terms with holes?

What **type** is synthesized for the function as a whole?



```
fun summary_stats(m : matrix) =  
  { mean    = stats.mean(m, ColumnWise),  
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```

```
matrix →  
{ mean    : vec,  
  std     : vec,  
  median  : □ }
```


Q: How to reason statically about terms with type errors?


What **type** is synthesized for the function as a whole?



```
fun summary_stats(m : matrix) =  
  { mean    = stats.mean(m, ColumnWise),  
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                                ▲
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Reify type inconsistencies as non-empty holes!

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Reify type inconsistencies as non-empty holes!

```
matrix →  
{ mean    : vec,  
  std     : vec,  
  median  : □ }
```

Contribution 1: A static semantics for lambda terms with holes

HTyp $\dot{\tau} ::= (\dot{\tau} \rightarrow \dot{\tau}) \mid \mathbf{num} \mid \langle \rangle$

HExp $\dot{e} ::= x \mid (\lambda x. \dot{e}) \mid \dot{e}(\dot{e}) \mid \underline{n} \mid (\dot{e} + \dot{e}) \mid \dot{e} : \dot{\tau} \mid \langle \rangle \mid \langle \dot{e} \rangle$

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$\boxed{\dot{\Gamma} \vdash \dot{e} \Rightarrow \dot{\tau}}$ \dot{e} synthesizes $\dot{\tau}$

$\boxed{\dot{\Gamma} \vdash \dot{e} \Leftarrow \dot{\tau}}$ \dot{e} analyzes against $\dot{\tau}$

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...

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$\frac{\dot{\Gamma} \vdash \dot{e} \Rightarrow \dot{\tau}}{\dot{\Gamma} \vdash \langle \dot{e} \rangle \Rightarrow \langle \rangle}$

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...

$\frac{\dot{\Gamma} \vdash \dot{e} \Rightarrow \dot{\tau}' \quad \dot{\tau} \sim \dot{\tau}'}{\dot{\Gamma} \vdash \dot{e} \Leftarrow \dot{\tau}}$

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$\boxed{\dot{\tau} \sim \dot{\tau}'}$ $\dot{\tau}$ and $\dot{\tau}'$ are consistent

$\overline{\langle \rangle \sim \dot{\tau}} \quad \overline{\dot{\tau} \sim \langle \rangle} \quad \overline{\dot{\tau} \sim \dot{\tau}} \quad \frac{\dot{\tau}_1 \sim \dot{\tau}'_1 \quad \dot{\tau}_2 \sim \dot{\tau}'_2}{(\dot{\tau}_1 \rightarrow \dot{\tau}_2) \sim (\dot{\tau}'_1 \rightarrow \dot{\tau}'_2)}$

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...

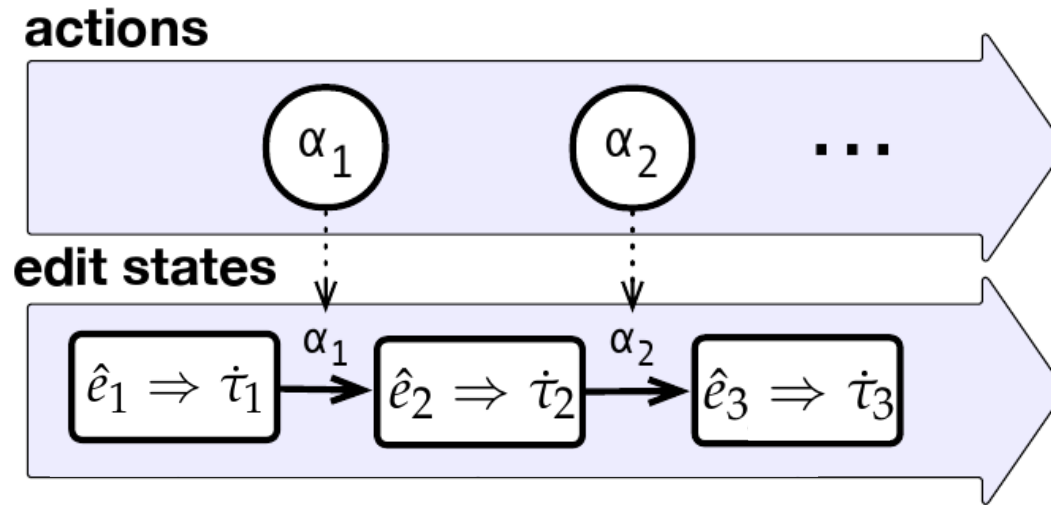
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coincides with **gradual typing**
[Siek and Taha, 2006]

Contribution 2: A typed edit action semantics



See <http://hazeltrove.org/>

Contribution 2: A typed edit action semantics

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Action $\alpha ::= \text{move } \delta \mid \text{construct } \psi \mid \text{del} \mid \text{finish}$
Dir $\delta ::= \text{child } n \mid \text{parent}$
Shape $\psi ::= \text{arrow} \mid \text{num}$
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$$\boxed{\dot{\Gamma} \vdash \hat{e} \Rightarrow \hat{\tau} \xrightarrow{\alpha} \hat{e}' \Rightarrow \hat{\tau}'}$$

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$$\frac{\hat{\tau} \approx \text{num}}{\dot{\Gamma} \vdash \triangleright \langle \rangle \triangleleft \xrightarrow{\text{construct lit } n} \langle \triangleright \underline{n} \triangleleft \rangle \Leftarrow \hat{\tau}}$$

Metatheorem: Sensibility

Every edit action leaves the edit state well-typed.

Theorem 1 (Action Sensibility).

1. If $\dot{\Gamma} \vdash \hat{e}^\diamond \Rightarrow \dot{\tau}$ and $\dot{\Gamma} \vdash \hat{e} \Rightarrow \dot{\tau} \xrightarrow{\alpha} \hat{e}' \Rightarrow \dot{\tau}'$ then $\dot{\Gamma} \vdash \hat{e}'^\diamond \Rightarrow \dot{\tau}'$.
2. If $\dot{\Gamma} \vdash \hat{e}^\diamond \Leftarrow \dot{\tau}$ and $\dot{\Gamma} \vdash \hat{e} \xrightarrow{\alpha} \hat{e}' \Leftarrow \dot{\tau}$ then $\dot{\Gamma} \vdash \hat{e}'^\diamond \Leftarrow \dot{\tau}$.

Metatheorem: Reachability

The cursor can reach any position in the program.

Theorem 3 (Reachability).

1. If $\hat{\tau}^\diamond = \hat{\tau}'^\diamond$ then there exists some $\bar{\alpha}$ such that $\bar{\alpha}$ movements and $\hat{\tau} \xrightarrow{\bar{\alpha}}^* \hat{\tau}'$.
2. If $\dot{\Gamma} \vdash \hat{e}^\diamond \Rightarrow \dot{\tau}$ and $\hat{e}^\diamond = \hat{e}'^\diamond$ then there exists some $\bar{\alpha}$ such that $\bar{\alpha}$ movements and $\dot{\Gamma} \vdash \hat{e} \Rightarrow \dot{\tau} \xrightarrow{\bar{\alpha}}^* \hat{e}' \Rightarrow \dot{\tau}$.
3. If $\dot{\Gamma} \vdash \hat{e}^\diamond \Leftarrow \dot{\tau}$ and $\hat{e}^\diamond = \hat{e}'^\diamond$ then there exists some $\bar{\alpha}$ such that $\bar{\alpha}$ movements and $\dot{\Gamma} \vdash \hat{e} \xrightarrow{\bar{\alpha}}^* \hat{e}' \Leftarrow \dot{\tau}$.

Metatheorem: Constructability

Any well-typed expression can be constructed using edit actions.

Theorem 6 (Constructability).

1. For every $\dot{\tau}$ there exists $\bar{\alpha}$ such that $\triangleright(\parallel)\triangleleft \xrightarrow{\bar{\alpha}}^* \triangleright\dot{\tau}\triangleleft$.
2. If $\dot{\Gamma} \vdash \dot{e} \Rightarrow \dot{\tau}$ then there exists $\bar{\alpha}$ such that:

$$\dot{\Gamma} \vdash \triangleright(\parallel)\triangleleft \Rightarrow (\parallel) \xrightarrow{\bar{\alpha}}^* \triangleright\dot{e}\triangleleft \Rightarrow \dot{\tau}$$

3. If $\dot{\Gamma} \vdash \dot{e} \Leftarrow \dot{\tau}$ then there exists $\bar{\alpha}$ such that:

$$\dot{\Gamma} \vdash \triangleright(\parallel)\triangleleft \xrightarrow{\bar{\alpha}}^* \triangleright\dot{e}\triangleleft \Leftarrow \dot{\tau}$$

Summary: Hazelnut

- A **static semantics** for terms with holes and type inconsistencies.
- An **typed action semantics** that maintains sensibility invariant.
 - **HZ**: A reference implementation written in OCaml React + js_of_ocaml.
- A **rich metatheory** that establishes the correctness of Hazelnut.
 - Mechanized using the **Agda** proof assistant.
 - Guides the definition of an extension (sum types – see paper!)

From Hazelnut to Hazel



Hazel

Numerics ▾

Plotting ▾

Statistics ▾

+

```
fun summary_stats(m : matrix<float>)
```

```
  { mean = mean(m, ColumnWise)
```

```
    std = std(m, □)
```

```
    median = □
```

(a)

```
let my_data : matrix<float> =
```

1.1	2.3	3.0	4.1	5.2
1.2	1.8	3.1	4.1	5.2
0.9	2.2	2.7	3.5	4.9
0.8	1.5	3.3	4.3	4.7

(b)

```
summary_stats(my_data)
```

```
  { mean = [1.0 | 2.0 | 3.0 | 4.0 | 5.0]
```

```
    std = std(my_data, □)
```

```
    median = □
```

(c)

Type at cursor: dimension

Action search...

(d)

ColumnWise (most probable)

RowWise

Factor to variable...

□(□)

Full action palette...

From Hazelnut to Hazel



Hazel

Numerics ▾

Plotting ▾

Statistics ▾

+

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(a)

TODO: scale up POPL17

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1.2	1.8	3.1	4.1	5.2
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0.8	1.5	3.3	4.3	4.7

(b)

```
summary_stats(my_data)
```

```
{ mean = [1.0 | 2.0 | 3.0 | 4.0 | 5.0]
```

```
  std = std(my_data, □)
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  median = □
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(c)

Type at cursor: dimension

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ColumnWise (most probable)

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□(□)

Full action palette...

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```
let my_data : matrix<float> =  
  [ 1.1 | 2.3 | 3.0 | 4.1 | 5.2  
    1.2 | 1.8 | 3.1 | 4.1 | 5.2  
    0.9 | 2.2 | 2.7 | 3.5 | 4.9  
    0.8 | 1.5 | 3.3 | 4.3 | 4.7 ]
```

(b)

```
summary_stats(my_data)
```

```
{ mean    = [1.0 | 2.0 | 3.0 | 4.0 | 5.0]  
  std     = std(my_data, □)  
  median  = □ }
```

(c)

Type at cursor: dimension

Action search...

(d)

ColumnWise (most probable)

TODO: type-specific projections
(based on my work at ICSE 2012, ECOOP 2014)

□(□)

Full action palette...

From Hazelnut to Hazel



```
fun summary_stats(m : matrix<float>)  
  { mean    = mean(m, ColumnWise)  
    std     = std(m, □)  
    median  = □ }
```

(a)

```
let my_data : matrix<float> =  
  [ 1.1 | 2.3 | 3.0 | 4.1 | 5.2  
    1.2 | 1.8 | 3.1 | 4.1 | 5.2  
    0.9 | 2.2 | 2.7 | 3.5 | 4.9  
    0.8 | 1.5 | 3.3 | 4.3 | 4.7 ]
```

(b)

```
summary_stats(my_data)
```

```
{ mean    = [1.0 | 2.0 | 3.0 | 4.0 | 5.0]  
  std     = std(my_data, □)  
  median  = □ }
```

(c)

TODO: a dynamic semantics for incomplete programs (*very live programming*)

Type at cursor: dimension

Action search...

(d)

ColumnWise (most probable)

RowWise

Factor to variable...

□(□)

Full action palette...

From Hazelnut to Hazel



Hazel

Numerics ▾

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+

TODO: an action suggestion semantics

```
fun summary_stats(m : matrix<float>)
```

```
{ mean = mean(m, ColumnWise)
```

```
{ std = std(m, □)
```

```
{ median = □
```

(a)

```
let my_data : matrix<float> =
```

1.1	2.3	3.0	4.1	5.2
1.2	1.8	3.1	4.1	5.2
0.9	2.2	2.7	3.5	4.9
0.8	1.5	3.3	4.3	4.7

(b)

```
summary_stats(my_data)
```

```
{ mean = [1.0 | 2.0 | 3.0 | 4.0 | 5.0]
```

```
{ std = std(my_data, □)
```

```
{ median = □
```

(c)

Action search...

(d)

ColumnWise (most probable)

RowWise

Factor to variable...

□(□)

Full action palette...

From Hazelnut to Hazel



```
fun summary_stats(m : matrix<float>)  
  { mean    = mean(m, ColumnWise)  
    std     = std(m, □)  
    median  = □ }
```

(a)

```
let my_data : matrix<float> =  
  [ 1.1 | 2.3 | 3.0 | 4.1 | 5.2  
    1.2 | 1.8 | 3.1 | 4.1 | 5.2  
    0.9 | 2.2 | 2.7 | 3.5 | 4.9  
    0.8 | 1.5 | 3.3 | 4.3 | 4.7 ]
```

```
summary_stats(my_data)
```

```
{ mean    = [1.0 | 2.0 | 3.0 | 4.0 | 5.0]  
  std     = std(my_data, □)  
  median  = □ }
```

(c)

Type at cursor: dimension

Action search...

(d)

ColumnWise (most probable)

TODO: a statistical model of edit actions

□(□)

Full action palette...

From Hazelnut to Hazel



```
fun summary_stats(m : matrix<float>)  
  { mean    = mean(m, ColumnWise)  
    std     = std(m, □)  
    median  = □ } (a)
```

```
let my_data : matrix<float> =  
  [ 1.1 | 2.3 | 3.0 | 4.1 | 5.2  
    1.2 | 1.8 | 3.1 | 4.1 | 5.2  
    0.9 | 2.2 | 2.7 | 3.5 | 4.9  
    0.8 | 1.5 | 3.3 | 4.3 | 4.7 ] (b)
```

```
summary_stats(my_data)
```

```
{ mean    = [1.0 | 2.0 | 3.0 | 4.0 | 5.0]  
  std     = std(my_data, □)  
  median  = □ } (c)
```

Type at cursor: dimension

Action search...

ColumnWise (most probable)

RowWise

Factor to variable...

TODO: library-defined derived actions

From Hazelnut to Hazel



Hazel

Numerics ▾

Plotting ▾

Statistics ▾

+

```
fun summary_stats(m : matrix<float>)
```

```
  { mean    = mean(m, ColumnWise)
```

```
    std     = std(m, □)
```

```
    median = □
```

(a)

```
let my_data : matrix<float> =
```

1.1	2.3	3.0	4.1	5.2
1.2	1.8	3.1	4.1	5.2
0.9	2.2	2.7	3.5	4.9
0.8	1.5	3.3	4.3	4.7

(b)

```
summary_stats(my_data)
```

```
  { mean    = [1.0 | 2.0 | 3.0 | 4.0 | 5.0]
```

```
    std     = std(my_data, □)
```

```
    median = □
```

(c)

Type at cursor: dimension

Action search...

(d)

ColumnWise (most probable)

RowWise

Factor to variable...

□(□)

Full action palette...

From Hazelnut to Hazel



```
fun summary_stats(m : matrix<float>)  
  { mean    = mean(m, ColumnWise)  
    std     = std(m, □)  
    median  = □ }
```

(a)

```
let my_data : matrix<float> = 

|     |     |     |     |     |
|-----|-----|-----|-----|-----|
| 1.1 | 2.3 | 3.0 | 4.1 | 5.2 |
| 1.2 | 1.8 | 3.1 | 4.1 | 5.2 |
| 0.9 | 2.2 | 2.7 | 3.5 | 4.9 |
| 0.8 | 1.5 | 3.3 | 4.3 | 4.7 |


```

(b)

```
summary_stats(my_data)
```

```
{ mean    = [1.0 | 2.0 | 3.0 | 4.0 | 5.0]  
  std     = std(my_data, □)  
  median  = □ }
```

(c)

Type at cursor: dimension

Action search...

(d)

ColumnWise (most probable)

RowWise

Factor to variable...

□(□)

Full action palette...

See <http://www.hazelgrove.org/>.

Thanks!